

# Limits

$$1) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$$

$$\text{Soln } \lim_{x \rightarrow -1} \frac{(x-1)(x+1)}{(x+1)}$$

$$-1 - 1 = -2$$

$$2) \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 9}$$

$$\text{Soln } \lim_{x \rightarrow 3} \frac{(x-3)(x-3)}{(x+3)(x-3)}$$

$$\frac{3-3}{3+3} = \frac{0}{6} = 0$$

$$3) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$\text{Soln } \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)}$$

$$\frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

$$\text{OR } \lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x-9)(\sqrt{x} + 3)}$$

$$\lim_{x \rightarrow 9} \frac{(x-9)}{(x-9)(\sqrt{x} + 3)} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

$$4) \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$$

$$\text{Soln } \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$\lim_{h \rightarrow 0} \frac{4+h-4}{h(\sqrt{4+h} + 2)}$$

$$\lim_{h \rightarrow 0} \frac{h}{h(\sqrt{4+h} + 2)}$$

$$\frac{1}{2+2} = \frac{1}{4}$$

$$5) \lim_{x \rightarrow 2} \frac{\sqrt{4-4x+x^2}}{x-2}$$

$$\text{Soln } \lim_{x \rightarrow 2^-} \frac{\sqrt{(2-x)^2}}{x-2}$$

$$\lim_{x \rightarrow 2^+} \frac{|2-x|}{x-2}$$

$$\lim_{x \rightarrow 2^-} \frac{|2-x|}{x-2}$$

$$\lim_{x \rightarrow 2^+} \frac{2-x}{x-2} = 1$$

-1

$\therefore \lim_{x \rightarrow 2} f(x) = \text{D.N.E.}$

$$6) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$$

$$\text{soln: } \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{x+3-4}$$

$$\lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x-1)}}$$

$$\sqrt{1+3} + 2 = 4$$

$$7) \lim_{x \rightarrow -1} \frac{x^3+1}{x+1}$$

$$\text{soln: } \lim_{x \rightarrow -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}}$$

$$= (-1)^2 - (-1) + 1$$

$$1 + 1 + 1 = 3$$

$$8) \lim_{x \rightarrow 2} \frac{x^4-16}{x^3-8}$$

$$\text{soln: } \lim_{x \rightarrow 2} \frac{(x^2+4)(x-2)(x+2)}{\cancel{(x-2)}(x^2+2x+4)}$$

$$\frac{(2^2+4)(2+2)}{2^2+2(2)+4} = \frac{32}{12} = \frac{8}{3}$$

$$9) \lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$$

$$|x-2| = x-2 \text{ if } x \geq 2$$

$$|x-2| = -(x-2) \text{ if } x < 2$$

soln:

$$\lim_{x \rightarrow 2^+} \frac{x-2}{x-2} = 1$$

$$\lim_{x \rightarrow 2^-} \frac{-(x-2)}{x-2} = -1$$

$$\text{Since } \lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x)$$

no limit  
D.N.E.

$$10) a) \lim_{x \rightarrow 1^-} \frac{1}{x-1} \quad \boxed{-\infty}$$

$$b) \lim_{x \rightarrow 1^+} \frac{1}{x-1} \quad \boxed{\infty}$$

$$c) \lim_{x \rightarrow -2^-} \frac{1}{x+2} \quad \boxed{-\infty}$$

$$d) \lim_{x \rightarrow -2^+} \frac{1}{x+2} \quad \boxed{\infty}$$

$$e) \lim_{x \rightarrow 3^-} \frac{1}{3-x} \quad \boxed{+\infty}$$

$$f) \lim_{x \rightarrow 3^+} \frac{1}{3-x} \quad \boxed{-\infty}$$