The Axioms of a Vector Space

A set V together with addition "$\oplus$" and scalar multiplication "$\cdot$" is a **Vector Space** if and only if each of the following axioms hold:

A1. for every $\mathbf{u}, \mathbf{v} \in V$, $\mathbf{u} \oplus \mathbf{v} \in V$,

A2. for every $\mathbf{u}, \mathbf{v} \in V$, $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$,

A3. for every $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$, $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$,

A4. there exists an element $\mathbf{0} \in V$ such that for every $\mathbf{u} \in V$, $\mathbf{0} \oplus \mathbf{u} = \mathbf{u}$,

A5. for every $\mathbf{u} \in V$, there exists a "$-\mathbf{u}$" $\in V$ such that $\mathbf{u} \oplus (-\mathbf{u}) = \mathbf{0}$,

M1. for every $\mathbf{u} \in V$ and $k \in \mathbb{R}$, $k \cdot \mathbf{u} \in V$,

M2. for every $\mathbf{u}, \mathbf{v} \in V$ and $k \in \mathbb{R}$, $k \cdot (\mathbf{u} \oplus \mathbf{v}) = k \cdot \mathbf{u} \oplus k \cdot \mathbf{v}$,

M3. for every $\mathbf{u} \in V$, and $k, m \in \mathbb{R}$, $(k + m) \cdot \mathbf{u} = k \cdot \mathbf{u} \oplus m \cdot \mathbf{u}$,

M4. for every $\mathbf{u} \in V$, and $k, m \in \mathbb{R}$, $k \cdot (m \cdot \mathbf{u}) = (km) \cdot \mathbf{u}$, and

M5. for every $\mathbf{u} \in V$, $1 \cdot \mathbf{u} = \mathbf{u}$.