

DATE: May 31, 2010

MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: MATH 1300

TITLE PAGE

EXAMINATION: Vector Geometry and Linear AlgebraTIME: 1 hourEXAMINER: Borgersen

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No texts or notes are permitted. No calculators are permitted. Cell phones, electronic translators, and other electronic devices are **not** permitted.

This exam has a title page and 8 pages of questions, including 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated beside the statement of the question. The total value of all questions is 60 points.

If you need more scrap paper, use the back of the question pages.

| Question | Points | Score |
|----------|--------|-------|
| 1 | 8 | |
| 2 | 10 | |
| 3 | 4 | |
| 4 | 4 | |
| 5 | 8 | |
| 6 | 10 | |
| 7 | 6 | |
| 8 | 10 | |
| Total: | 60 | |

Short Answer

1. [8 points] Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 1 & -3 \end{bmatrix}.$$

In each of the following cases, compute the given expression or briefly explain why the expression cannot be calculated:

(a) AB (b) $A + B$ (c) $B + 2C^T$ (d) $AB - BA$

DATE: May 31, 2010

MIDTERM EXAMINATION

PAGE: 2 of 8

DEPARTMENT & COURSE NO: MATH 1300TIME: 1 hourEXAMINATION: Vector Geometry and Linear AlgebraEXAMINER: Borgersen

2. Let A be a 3×3 matrix with determinant 5, and let B be a 3×3 matrix with determinant -3 . Find the determinant of each of the following (showing all necessary work):

(a) [3 points] AB^2

(b) [3 points] $A^{-1}(2B)A^T$

(c) [4 points] $\text{adj}(A)$

3. [4 points] Suppose A is a 4×4 invertible matrix.

(a) What is the reduced row echelon form of A ?

(b) Find all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$.

4. Consider the system of linear equations

$$\begin{aligned}x + ay &= 2 - b \\4x + 2ay &= 2b\end{aligned}$$

This system's augmented matrix is $\left[\begin{array}{cc|c} 1 & a & 2 - b \\ 4 & 2a & 2b \end{array} \right]$, which, partially reduced is $\left[\begin{array}{cc|c} 1 & 0 & -2 + 2b \\ 0 & a & 4 - 3b \end{array} \right]$.

(a) [2 points] Find all a and b such that the system has no solutions.

(b) [2 points] Find all a and b such that the system has infinitely many solutions.

Long Answer

5. [8 points] Let A be some fixed square matrix. Let B and C be two inverses of A . Prove then that $B = C$ (that is, that there is only one inverse of A). Show all necessary steps, and use complete sentences.

6. Let

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ -2 & 2 & 2 \end{bmatrix}.$$

The adjoint of A is shown below partially computed.

(a) [4 points] Enter the two missing numbers in the boxes provided.

$$\text{adj}(A) = \begin{bmatrix} -4 & -4 & \square \\ \square & -4 & -4 \\ 4 & -8 & -4 \end{bmatrix}.$$

(b) [4 points] Find the determinant of A (using any valid method)

(c) [2 points] Use (a) and (b) above to find A^{-1} .

7. [6 points] Solve the following system using Gauss-Jordan Elimination. No marks will be awarded for any other method.

$$\begin{aligned}x_1 & \quad \quad + 2x_3 + x_4 = 10 \\-x_1 + x_2 + x_3 - x_4 & = -5 \\x_1 + x_2 + 5x_3 + 2x_4 & = 21 \\& \quad \quad x_2 + 3x_3 = 5\end{aligned}$$

DATE: May 31, 2010

MIDTERM EXAMINATION

PAGE: 6 of 8

DEPARTMENT & COURSE NO: MATH 1300TIME: 1 hourEXAMINATION: Vector Geometry and Linear AlgebraEXAMINER: Borgersen

8. [10 points] Express $A = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$ as a product of elementary matrices. Show all your work.

UNIVERSITY OF MANITOBA

DATE: May 31, 2010

MIDTERM EXAMINATION

PAGE: 7 of 8

DEPARTMENT & COURSE NO: MATH 1300

TIME: 1 hour

EXAMINATION: Vector Geometry and Linear Algebra

EXAMINER: Borgersen

SCRAP PAPER

UNIVERSITY OF MANITOBA

DATE: May 31, 2010

MIDTERM EXAMINATION

PAGE: 8 of 8

DEPARTMENT & COURSE NO: MATH 1300

TIME: 1 hour

EXAMINATION: Vector Geometry and Linear Algebra

EXAMINER: Borgersen

SCRAP PAPER