MATH 1500 Quiz 4 Solutions 10S Term 1

Name: ______________________________ Student Number: __________________

Answer all questions and show all your work. (Total Marks: 20).
You have 20 minutes to complete the quiz.

[4] 1. Find all the critical numbers of $B(x)$ given that

$$B(x) = 3x^{2/3} - x \qquad \text{and} \qquad B'(x) = \frac{2 - \sqrt[3]{x}}{\sqrt[3]{x}}.$$

Solution:

1. First, when $B'(x) = 0$:

$$\frac{2 - \sqrt[3]{x}}{\sqrt[3]{x}} = 0 \implies 2 - \sqrt[3]{x} = 0 \implies \sqrt[3]{x} = 2 \implies x = 8.$$

2. Second, when $B(x)$ exists, but $B'(x)$ doesn't. The domain of $B(x)$ is $\mathbb{R}$. The domain of $B'(x)$ is $(-\infty, 0) \cup (0, \infty)$. Therefore 0 is also a critical number.

Thus the critical numbers of $B(x)$ are 0 and 8.

[4] 2. A function $h(x)$ is continuous on $(-\infty, \infty)$. The critical numbers of $h(x)$ are -3, 0, and 1. The following values are calculated:

$$h(-4) = 10, \quad h(-3) = -7, \quad h(-2) = 0, \quad h(-1) = 4, \quad h(0) = 5,$$

$$h(1) = 3, \quad h(2) = 0, \quad h(3) = -10 \quad h(4) = -10.$$

Find the absolute max and min values of $h(x)$ on the interval $[-2, 3]$.

Solution: We need to check the end points of the interval, and all critical numbers in it.

$$h(-2) = 0 \quad h(0) = 5 \quad h(1) = 3 \quad h(3) = -10$$

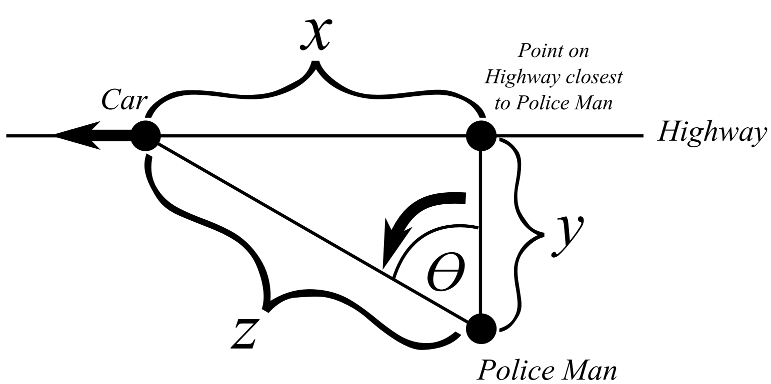
The absolute max occurs at $x = 0$, and the absolute min occurs at $x = 3$.

[2] 3. Suppose $g(x)$ is a function such that $g''(x)$ is continuous on $(-\infty, \infty)$. If $g'(2) = 0$ and $g''(2) = -5$, what can you say about g?

Solution: Since $g'(2) = 0$, 2 is a critical number of $g(x)$. Since $g''(2) = -5 < 0$, g is concave down at 2, and thus by the second derivative test, g has a local max at $x = 2$.

[10] 4. A policeman is standing near a highway using a radar gun to catch speeders. He aims the gun at a car that has just passed his position, and when the gun is pointing at an angle of 45 degrees to the direction of the highway, notes that the distance between the car and the gun is increasing at a rate of 100 km/h. How fast is the car traveling?

Solution: We have the following picture:



We are asked to find $\frac{dx}{dt}$ at a particular moment in time when $\theta = \frac{\pi}{4}$, and $\frac{dz}{dt} = 100$. We have one main relationship of importance:

$$x^2 + y^2 = z^2.$$

Taking the derivative of both sides with respect to time, we get

$$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$$

(the derivative of y^2 is zero, since the distance y is a constant–that is, it is not changing over time). Solving for $\frac{dx}{dt}$, we get

$$\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}.$$

The above formula is true at all time. Now we look at the particular time, when $\theta = \pi/4$ and $\frac{dz}{dt} = 100$. The only thing left to realize to solve the problem is, even though we do not know either z or x, we do know that

$$\frac{z}{x} = \frac{1}{\sin\theta} = \frac{1}{\sin(\pi/4)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Therefore, at this particular time,

$$\frac{dx}{dt} = \sqrt{2}(100) = 100\sqrt{2}.$$

Therefore the car is traveling at $100\sqrt{2}$ km/hr.