Review

Use limits to determine the values of k for which the following function is continuous at $x=5$.

$$f(x) = \begin{cases} 2k - x & \text{if } x \le 5 \\ x^2 + kx & \text{if } x > 5 \end{cases}$$

tell th

[$f(x)$ consists of polynomials ~~but~~ ∴ continuous on respective domain, but asking only at $x=5$]

Answer

to be continuous at $x = 5$ } they must write

$$\lim_{x \to 5^-} 2k - x = f(5) = \lim_{x \to 5^+} x^2 + kx$$

$$\lim_{x \to 5^-} 2k - x = 2k - 5 = f(5)$$

$$\lim_{x \to 5^+} x^2 + kx = 25 + 5k$$

$$2k - 5 = 25 + 5k$$

$$-30 = 3k$$

$$-10 = k$$

sub in for k + x

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Find k to make $f(x)$ continuous everywhere

$$f(x) = \begin{cases} x+1 & x \le 1 \\ 2x^2 - x + k & x > 1 \end{cases}$$

Find k to make $f(x)$ contenuous everywhere.

$$f(x) = \begin{cases} x+1 & x \le 1 \\ 2x^2 - x + k & x > 1 \end{cases}$$

they must ~~say~~ write

Since $f(x)$ consists of polynomials, it is contenuous everywhere on its domain,
Possible discontenuity at $x=1$

they must ~~say~~ write

to be contenuous:

$$\lim_{x \to 1^-} x+1 = \lim_{x \to 1^+} 2x^2 - x + k = f(1)$$

$$\lim_{x \to 1^-} x+1 = 2 = f(1) \qquad \lim_{x \to 1^+} 2x^2 - x + k = 1+k$$

$\therefore$ to be contenuous everywhere $2 = 1+k$

$$\underline{1 = k}$$

If $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } 1 \le x \le 2 \\ x^3 - x & \text{if } x > 2 \end{cases}$$

determine whether $f(x)$ is continuous (a) $x = 1$ (b) $x = 2$

~~you say~~

$\left\{ \text{Since } f(x) \text{ is made up of polynomials, it is continuous everywhere on its respective domains with possible discontinuities at } x = 1 \text{ \& } x = 2 \right\}$

to be continuous at $x = 1$

$$\lim_{x \to 1^-} x^2 = \lim_{x \to 1^+} 2x - 1 = f(1)$$

$$\lim_{x \to 1^-} x^2 = 1 \qquad \lim_{x \to 1^+} 2x - 1 = 1 = f(1)$$

$\therefore$ $f(x)$ is continuous at $x = 1$

since $\lim_{x \to 1^-} x^2 = 1 = \lim_{x \to 1^+} 2x - 1 = f(1)$

to be continuous at $x = 2$

$$\lim_{x \to 2^-} 2x - 1 = f(2) = \lim_{x \to 2^+} x^3 - x$$

$$\lim_{x \to 2^-} 2x - 1 = 3 = f(2)$$

$$\lim_{x \to 2^+} x^3 - x = 6$$

Since $\lim_{x \to 2^-} 2x - 1 = f(2) \neq \lim_{x \to 2^+} x^3 - x$

fcn is not continuous at $x = 2$.

Suppose $f(x) = \begin{cases} ux^2-2 & x<2 \\ 4u-v & x=2 \\ 2x-v & x>2 \end{cases}$

Find all possible pairs of (u,v) such that f is continuous everywhere.

$$f(x) = \begin{cases} ux^2-2 & x<2 \\ 4u-v & x=2 \\ 2x-v & x>2 \end{cases}$$

For $x \neq 2$, f is defined by polynomials and so is continuous.

To be continuous at $x=2$

$$\lim_{x \to 2^-} ux^2-2 = f(2) = \lim_{x\to 2^+} 2x-v$$

$$\lim_{x\to 2^-} ux^2-2 = 4u-2$$

$$f(2) = 4u-v$$

$$\lim_{x\to 2^+} 2x-v = 4-v$$

$$\therefore \quad 4u-2 = 4u-v$$
$$-2 = -v$$
$$2 = v$$

and $$4u-v = 4-v$$

But $v=2$ $$4u=4$$
$$u=1$$

$f(x)$ is continuous everywhere if $(u,v) = (1,2)$